SOLUTION OF THE INVERSE PROBLEM OF THERMAL CONDUCTION. INTERPRETATION OF CALORIMETER MEASUREMENTS OF THERMAL FLUXES

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A general method for interpreting temperature measurements obtained by means of the sensing elements of calorimeters installed in actual structures for the two-dimensional plane and axisymmetric cases for any time dependence of the thermal flux is proposed.

In experiments, thermal fluxes in pipes and actual devices are measured by means of data units whose sensing elements are always in contact of some kind with the structure of the simulator or device. The behavior of temperature as a function of time is obtained as a result of such measurements.

Consider the following problem: it is necessary to determine the thermal flux as a function of time on the basis of the measurements performed. The heat leakage from the sensing element to the structure can be so large that calculation of thermal fluxes without considering this leakage is practically impossible. The estimate of this heat leakage constitutes a complex physical problem.

Exact theoretical solutions of the problem of determining thermal fluxes in actual structures for an arbitrary time dependence of the temperature have not yet been obtained.

In the presently available papers [2-4], the inverse problem of heat conduction is solved analytically on the basis of relationships between the thermal flux, the temperature, the coordinates, and the time. However, these solutions pertain only to simple cases.

If the thermophysical characteristics depend on the temperature, the following method of computer solution of the inverse problem of thermal conduction for a plate is proposed in [3]; using the method of straight lines, we calculate the temperature distribution along the thickness with respect to the surface temperature assigned in the experiment, while the thermal flux is calculated with respect to the temperature gradient at the surface. It has been suggested to use 10-15 equations for calculating the temperatures; the authors claim that the thermal fluxes can then be calculated with an accuracy of 0.5-1%. However, no calculations are given in the paper.

For actual calorimeters, the problem is not one-dimensional. Calorimeters are complex devices, which include many parts with unlike thermophysical characteristics. There is three-dimensional heat overflow.

In order to solve this problem most efficiently, we propose to take into account the three-dimensional heat overflow and the difference between the thermophysical characteristics of the parts. The proposed method is suitable for computer solutions.

The problem is solved for the case where the temperature is measured at the surface to which the thermal flux is supplied. However, solutions can be found also if the temperature is measured at another location.

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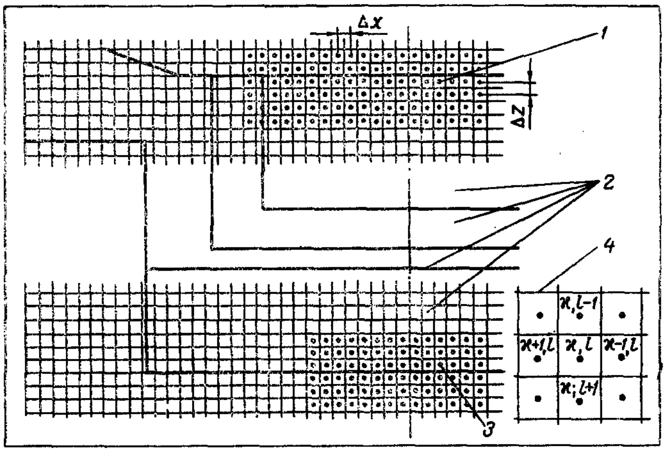

Fig. 1. Calculation scheme and indexing of points. 1) m_1 layer; 2) parts with unlike thermophysical characteristics (there can be six of them altogether); 3) m_2 layer; 4) indices of points.

The following initial data must be known in order to solve the problem:

1) the design of the flat or axisymmetric calorimeter or, in other words, the coordinates of contours of the calorimeter parts;

2) the thermophysical characteristics of the parts;

3) the table of the $T = f(\tau)$ functions for the point at which the thermal flux is measured;

4) the thermal flux conducted from within the frame;

5) the emissivities of the outside and inside surfaces of the calorimeter.

The calculations are performed in the following manner: the thermal flux is determined so that the value of $dT/d\tau$ determined from the thermal conduction equation for the point where the thermal flux is measured at a given instant of time coincides with the value of $dT/d\tau$ for the function $T = f(\tau)$ given in tabular form. Then, the heating of the calorimeter and the structural unit connected to it is calculated with respect to the determined thermal fluxes by means of the usual finite-difference equations of thermal conduction.

For the two-dimensional case, the thermal conduction equation has the following form [1]:

$$\frac{\partial T}{\partial \tau} = a\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{x}\cdot\frac{\partial T}{\partial x}S\right], \tag{1}$$

where $S = 0$ for the plane case, and $S = 1$ for the axisymmetric case.

For programmed calculations, we express the partial derivatives with respect to x and z in finite-difference form:

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{\varkappa,l-1} - 2T_{\varkappa,l} + T_{\varkappa,l+1}}{\Delta x^2};\quad \frac{\partial T}{\partial x} = \frac{T_{\varkappa,l+1} - T_{\varkappa,l-1}}{2\Delta x};$$
$$\frac{\partial^2 T}{\partial z^2} = \frac{T_{\varkappa-1,l} - 2T_{\varkappa,l} + T_{\varkappa+1,l}}{\Delta z^2}; \tag{2}$$

where Δx is the grid spacing along the x axis, and Δz is the grid spacing along the z axis.

The calculation scheme (grid) and the indexing of points are shown in Fig. 1.

For the external calorimeter elements m_1 and m_2 (see Fig. 1), which receive or release convective or radiant thermal fluxes, the working thermal conduction equation is given by

$$\frac{\partial T}{\partial \tau} = \left[\frac{\partial^2 T}{\partial x^2} + \frac{1}{x}\cdot\frac{\partial T}{\partial x}S + \frac{\partial^2 T}{\partial z^2}\right]a + \frac{p\alpha(T_l - iT) - \varepsilon\sigma\left(\frac{T}{100}\right)^4 j + kq_{\text{rad}}}{c\gamma\Delta h}, \tag{3}$$

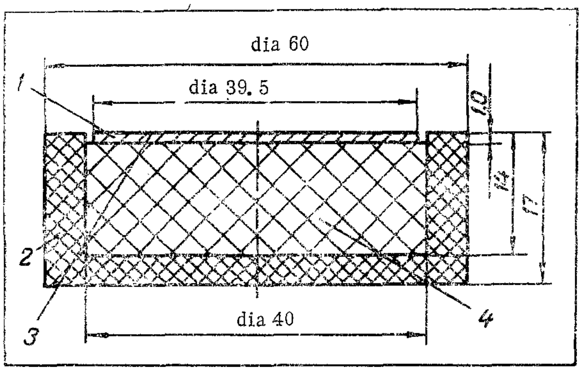

Fig. 2. Results of calculations of the thermal fluxes supplied to the solid. 1) Sensing element (steel); 2) insulator frame (a = 0.155 · 10^{-7} m^2/sec); 3) surface to which the thermal flux is supplied; 4) insulator (a = 0.155 · 10^{-7} m^2/sec).

where p, i, j, and k are coefficients, which are equal to 0 or 1, depending on the type of the boundary conditions, and Δh is the thickness of the layer of external elements.

The partial derivatives are also calculated by means of the finite-difference equations given above. The only difference is that, for the top elements m_1, $T_{\varkappa-1,l} = T_{\varkappa,l}$ if they are located on a horizontal straight line, and $T_{\varkappa,l+1} = T_{\varkappa,l}$ or $T_{\varkappa,l-1} = T_{\varkappa,l}$ if they are located on a vertical straight line, depending on the direction from which the thermal flux is supplied.

Similar relationships hold for the bottom elements m_2. It is assumed that $\lambda(\partial T/\partial x)$ vanishes at the axis.

The curves along which the thermal flux is supplied can have any shape.

The temperature at a certain instant of time at a given point is calculated by means of

$$T_{i+1} = T_i + \left(\frac{\partial T}{\partial \tau}\right)_i \Delta\tau, \tag{4}$$

where $\Delta\tau$ is the time interval.

The thermal fluxes are calculated when, in the process of calculation, the x and z coordinates are equal to the coordinates of the point for which the T = f(T) table is assigned. The value of $dT/d\tau$ is calculated with respect to this table:

$$\frac{dT}{d\tau} = \left(\frac{\partial T}{\partial \tau}\right)_{mp} = \frac{T_{j+1} - T_j}{\tau_{j+1} - \tau_j}, \tag{5}$$

where j pertains to the nodal points at which the $T = f(\tau)$ graph is assigned; the symbol mp denotes the points of temperature measurement. The time for which the calculations are performed lies within τ_{j+1} and τ_j.

The thermal flux is then determined by means of the expression

$$q_{\Sigma i} = c\gamma\Delta h \left\{ \left(\frac{\partial T}{\partial \tau}\right)_{mp} - \left[\frac{\partial^2 T}{\partial x^2} + \frac{1}{x} \cdot \frac{\partial T}{\partial x} S + \frac{\partial^2 T}{\partial z^2}\right] a \right\}, \tag{6}$$

i.e., the Laplacian pertains to the preceding instant of time. Since the time interval is very short in these calculations, the error in calculating the thermal fluxes is small, as we shall see later. In order to determine more accurately the thermal fluxes in solving the inverse problem, the author has introduced "feedback" in a differently composed program.

The thermal fluxes calculated by means of (6) increase or decrease until the calculated temperature and the temperature assigned at a given point coincide with the assigned temperature. However, this prolongs the calculation time.

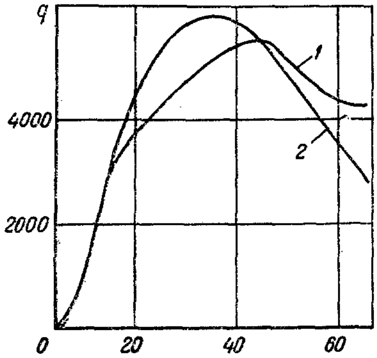

Fig. 3

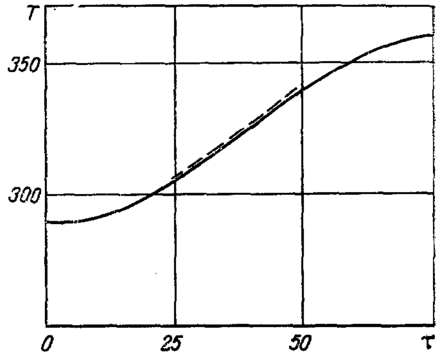

Fig. 4

Fig. 3. Thermal flux q (W/m^2) calculated with and without an allowance for the heat leakage as a function of the time τ (sec). 1) Computer interpretation of the thermal flux; 2) calculation of the thermal flux with respect to the heating rate of the data unit's sensing element without an allowance for leakage: $q = c\gamma\delta(dT/d\tau)$.

Fig. 4. Assigned and calculated functions at the point where the thermal flux is measured. The solid $T = f(\tau)$ curve pertains to measurement at the surface (the curve in tabular form is fed into the computer). The dashed $T = f(\tau)$ is obtained by calculation from the solution of the thermal conduction equation with respect to the thermal fluxes computed according to the program. T is the temperature (deg K); τ is the time (sec).

As an example of computer calculations, we provide the results obtained in calculating the thermal fluxes supplied to the solid shown in Fig. 2. The thermal fluxes calculated with an allowance for heat leakages (according to the program) and without taking into account the leakages are shown in Fig. 3. Figure 4 shows the assigned and the theoretical functions for the point at which the thermal flux is measured. They are in good mutual agreement.

The usual procedure is used in calculating the heating by means of the finite-difference method, and, therefore, the calculations of the thermal fluxes are undoubtedly correct.

It should be noted that no oscillation or nonconvergence of the solution was observed in any of the cases (of course, the value of $\Delta\tau$ was smaller than the well-known theoretical ratios $\Delta x^2/2a$ and $\Delta y^2/2a$ in the entire field of calculations; otherwise, oscillation would have occurred).

In conclusion, it should be mentioned that these results are of great importance for thermal flux measurements in tests and, consequently, in designing actual devices.

NOTATION

T	is the temperature;
x and z	are the coordinates;
q	is the thermal flux;
α	is the heat transfer coefficient;
T_l	is the recovery temperature;
τ	is the time;
c	is the specific heat;
γ	is the specific weight;
λ	is the thermal conductivity;
a	is the thermal diffusivity;
ε	is the emissivity.

LITERATURE CITED

1. A. V. Lykov, Heat Conduction Theory [in Russian], GITTL, Moscow (1952).
2. G. Carslow and D. Eger, Thermal Conductivity of Solids [Russian translation], Inostr. Lit., Moscow (1964).
3. G. G. Aldoshin, A. S. Golosov, and V. I. Zhuk, Heat and Mass Transfer [in Russian], Vol. 8, Nauka i Tekhnika, Minsk (1968).
4. O. N. Kastelin and L. N. Bronskii, in: Physical Gas Dynamics, Heat Exchange, and High-Temperature Gas Thermodynamics [in Russian], Izd. AN SSSR, Moscow (1962).